The gravitational wave background from neutron star formation and bar-mode instabilities

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Abstract. We present calculations of the stochastic gravitational wave background resulting from neutron star birth throughout the Universe, including order-of-magnitude estimates for post-collapse bar-mode instabilities based on simulations by Brown (2000) and Shibata et al. (2002). We employ three waveforms from Dimmelmeier, Font and Müller (2002) based on models, incorporating general relativistic effects, for the axisymmetric core collapse of rotating massive stars. Source-rate evolution is accounted for by using a star formation rate simulation based on a 'flat-Λ' cosmology (Hernquist & Springel 2003). We find that the core-collapse background signal is not detectable by cross correlating two Advanced LIGO detectors, but a background generated by bar-mode instabilities is potentially detectable in 1 year of integration time.

1. Introduction

While discovery of a stochastic gravitational wave (GW) background of primordial origin (Carr 1980) would provide a unique insight into the very early Universe, the detection of a stochastic background of astrophysical origin — from core-collapse supernovae (SNe) or other sources distributed throughout the cosmos — would open a new window to the high-redshift Universe. Predicting how these two very different backgrounds will manifest in GW detectors is an important and challenging task.

Ferrari, Matarrese & Schneider (1999a,b; hereafter FMSa,b) calculated the stochastic background resulting from black hole formation in core-collapse SNe and the background produced by the GW-driven instabilities called r-modes in rapidly rotating nascent neutron stars. Coward, Burman & Blair (2001; hereafter CBB) used a sample of numerical models for the axisymmetric rotational core collapse of massive stars (Zwerger & Müller 1997) to calculate the GW background resulting from core-collapse events that lead to the formation of neutron stars (NSs), and to develop an algorithm that simulates the background (Coward et al. 2002a,b); to account for source rate evolution, they used an observation-based star formation rate (SFR) model corrected for dust extinction, combined with the Einstein-de Sitter cosmology.

In this paper we revise that work to determine the spectral properties of the GW background using three recently computed general relativistic waveforms from the catalogue of Dimmelmeier, Font & Müller (2002; hereafter DFM), along with a more recent SFR model and the currently popular 'flat-Λ' cosmology.

2. The differential NS formation rate

As the lifetimes of stars more massive than (8–10) $M_\odot$ are only tens of Myr, the evolving rate of core-collapse SNe closely tracks the SFR. To determine the source-rate evolution we have employed the recent simulation-based SFR model of Hernquist and Springle (2003), developed in a 'flat-Λ' cold dark matter cosmology. This model, which includes the effects of feedback processes within the interstellar medium, is compatible with observation-based models of SFR evolution and is supported by recent data from the Wilkinson Microwave Anisotropy Probe suggesting that star formation started at $z > 9.5$ (Yokoyama 2003). The model SFR density rises exponentially from high z, reaches a peak at $z \sim 5-6$ and declines toward $z = 0$ as a power-law function of the Hubble parameter $H(z)$.

Using a local NS formation rate density $r_0^{\rm NS} \approx 5 \times 10^{-12}$ NS s^{-1} Mpc^{-3} (see CBB), we convert the SFR evolution model to a dimensionless evolution factor $e(z)$, with $e(0) = 1$. The variation of the NS formation rate with redshift is given by

$$dR^{\rm NS}/dz = 4\pi(c^3 {r_0}^{\rm NS}/{H_0}^3)e(z)F(z)/(1+z)\,, \tag{1}$$

where $R^{\rm NS}\,(z)$ is the all-sky event rate, as observed in our local frame, for sources out to redshift z (CBB). The dimensionless function $F(z)$ is determined by the cosmological model (Peebles 1993, p. 332); H_0 is the Hubble constant at the present epoch.

3. Calculating the GW background

The DFM waveforms are classified according to three different types of core-collapse events. Type I waveforms, from a 'regular collapse model', are characterized by a distinct spike, the result of core bounce, followed by a damped oscillation. Type II waveforms are typified by several distinct spikes due to multiple core bounces. A very rapid core collapse will result in a Type III waveform, which shows no apparent spike but is characterized by a maximum amplitude occurring at the first positive peak, followed by smaller negative and positive amplitudes pre- and post-bounce.

Bar modes associated with NS formation are potentially strong sources of GWs and could dominate the total GW emission associated with NS birth: recent calculations indicate that the resulting GW fluence could be orders of magnitude greater than that from the collapsing core. Based on the numerical bar-mode simulations of Brown (2000) and Shibata et al. (2002), we assume a (dimensionless) characteristic GW amplitude $h_{\rm c} \approx 10^{-22}$, a characteristic frequency $f_{\rm c} \approx 500$–600 Hz, and a total energy emission $\triangle E/M_\odot c^2 \approx 10^{-4}$. Although uncertain, these values are comparable to those from other models (e.g. Houser 1998, Saijo et al. 2001).

The spectral fluence, $F_{\rm ss}$, in J m^{-2} Hz^{-1}, of a single source is obtained using the methods of FMS and CBB. The background spectral flux density, in W m^{-2} Hz^{-1}, from all NS births throughout the Universe is obtained by integrating over the redshift range $z = 0$ to 10:

$$F_B(f) = \int_0^{10} [F_{\rm ss}(f,z) dR^{\rm NS}/dz]\; dz\,; \tag{2}$$

the observed frequency f is related to the source frequency f_s by the redshift factor: $f = f_s/(1+z)$. The background spectral strain, in Hz$^{-1/2}$, is calculated directly from F_B (FMSa,b):

$$\sqrt{S_B(f)} = \left(2G/\pi c^3\right)^{1/2} f^{-1}\left[F_B(f)\right]^{1/2}\,. \tag{3}$$

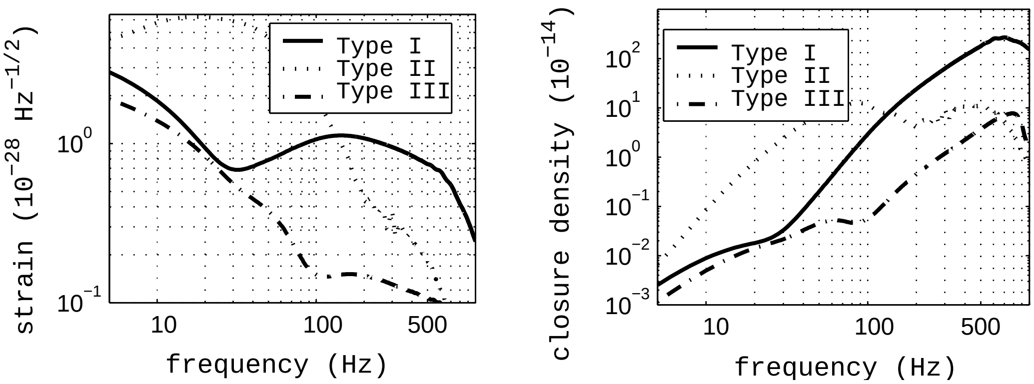

Figure 1. **(a)** The background spectral strain for the three relativistic waveforms. **(b)** The corresponding closure densities.

The spectral energy density of a GW background is conventionally expressed by the dimensionless 'closure density' $\Omega_B(f)$, defined as the energy density of GWs per logarithmic frequency interval normalized to the cosmological critical energy density $\rho_c c^2$; it also can be obtained from F_B (FMSa,b):

$$\Omega_B(f) = f[F_B(f)/(\rho_c c^3)]\,. \tag{4}$$

4. Numerical results

The integrated rate of NS formation, $R^{\rm NS}$, reaches an asymptotic value of about 25 $\rm s^{-1}$ out to $z = 10$. This result is similar to the previous calculation of $R^{\rm NS}$ (CBB), despite here using a SFR model biased to higher-z sources and extending the cutoff for NS formation from $z = 5$ to $z = 10$.

Figure 1(a) shows the spectral strain for backgrounds formed from each of the three selected relativistic waveforms. Prominent features in the single-source spectral strain for the three waveforms have been broadened and shifted to lower frequencies by a factor of about 5 — a result of the maximum in $\mathrm{d}R^{\rm NS}/\mathrm{d}z$ occurring near $z = 4$.

The low-frequency content displayed by this background suggests that a detector operating between the LISA and Advanced LIGO frequency bands, in (10^{-2}–10) Hz, may be advantageous. Although no detectors are planned within this frequency range, Seto et al. (2001) have discussed the possibility.

The corresponding closure densities are shown in figure 1(b). The Type I background has a maximum of about 10^{-12} at 700 Hz. The Type II and III peaks are an order of magnitude lower, at about 10^{-13}, near 100 Hz and 800 Hz respectively.

The background spectral strain and closure density are smaller than the results of CBB by at least an order of magnitude. These differences are attributable to relativistic effects in the collapse, which reduce the amplitude of the single-source waveforms, and the new SFR model, which predicts that 75% of stars were formed by $z = 1$, implying little recent star formation; at high z, contributions to the background are strongly reduced by the inverse luminosity-distance dependence.

Estimates of the duty cycle (DC) are in the range 0.3–0.7 for backgrounds consisting predominantly of Type I and III relativistic waveforms, implying a non-

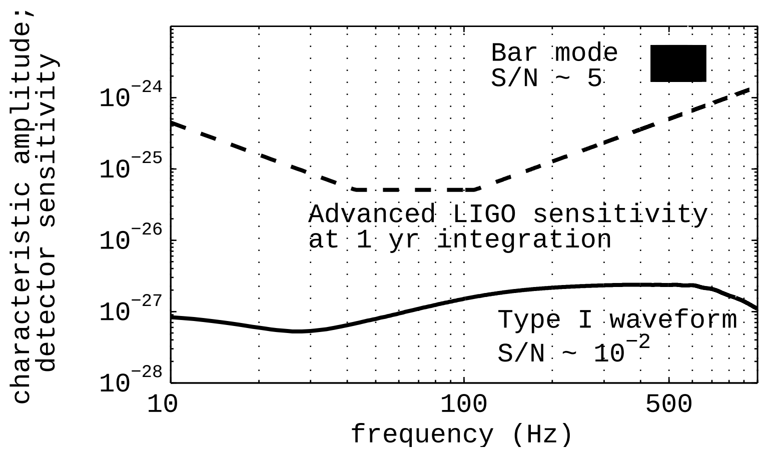

Figure 2. Background characteristic amplitudes for GW emission from NS formation and bar modes compared with detector sensitivity.

continuous, 'popcorn noise', signal. These are within the range of values calculated by CBB. The Type II waveform gives a continuous DC value of 3, the result of significant low-frequency content in this model.

To estimate the detectability of the background, the Type I $\Omega_B(f)$ shown in figure 1(b) is converted to a characteristic amplitude (Maggiore 2000, Section 2). The equivalent characteristic noise amplitude, assuming two Advanced LIGO detectors, is used to estimate the detector sensitivity after 1 year integration; this is based on the piecewise-parametrized model of Flanagan & Hughes (1998) for the detector noise. We find signal-to-noise ratio values of order 10^{-2} for the Type I core-collapse model, 10^{-3} for the Types II and III, and a value of about 5 for a bar-mode background (see figure 2) suggesting that, because of post-collapse instabilities, the GW background of the cosmological population of core-collapse SNe is potentially detectable by cross-correlating Advanced LIGO detectors.

In order to differentiate between this background and the stochastic background from other astrophysical sources, it will be necessary to statistically characterize the signal. Our present aim is to develop an optimized signal processing strategy that utilizes the distinctive GW amplitude distribution for backgrounds with a DC of order unity (Coward et al. 2002b).

There remains considerable uncertainty in our understanding of the core-collapse process that leads to NS formation. Observational evidence suggests that some of the neutron stars receive kick velocities of order 1000 km/s. A systematic study of supernova explosions by Scheck et al. (2003) using hydrodynamic simulations yields evidence that neutrino-driven convection behind the expanding supernova shock can lead to global asymmetries consistent with observed pulsar kick velocities. This indicates that the GW emission models used here may be an underestimate, at least for a fraction of events.

Acknowledgments

We gratefully thank H. Dimmelmeier, J. A. Font and E. Müller for the use of their waveform data and J. Brown and M. Shibata for providing simulated bar-mode data and for useful discussions. We also thank the referees for helpful comments. This research is supported by the Australian Research Council Discovery Grant DP0346344 and is part of the research program of the Australian Consortium for Interferometric Gravitational Astronomy (ACIGA). E. Howell gratefully acknowledges support from ACIGA and D. Coward is supported by an Australian Post Doctoral Fellowship.

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